

Dark Matter Strikes Back

Paolo Salucci

SISSA, Via Bonomea 265, 34012 Trieste, Italy

(Dated: February 3, 2017)

Mc Gaugh et al. (2016) have found, by investigating a large sample of Spirals, a tight non linear relationship between the total radial acceleration, connected with the Dark Matter phenomenon, and its component which comes from the distribution of baryonic matter, as the stellar and HI disks. The strong link between these two quantities is considered by them and by other researchers, as challenging the scenario featuring the presence of DM halos in galaxies. Or, at least, to indicate the peculiar nature of the underlying dark matter particles. We have explored this issue by investigating a larger number of galaxies by means of several techniques of analysis. Our results support and even increase, both qualitatively and quantitatively, the validity of McGaugh et al. (2016) 's relationship. However, we prove that such relationship exists also in the scenario featuring dark matter halos + ordinary baryonic matter and that it arises by the fact the DM is less concentrated than the luminous matter and it is progressively more abundant in lower luminosity objects. These properties are due to well known astrophysical effects: the implications of this relationship for the properties of dark matter halos are nothing of new or of unexpected. The relationship, definitively, is not a portal to go beyond the standard picture of Λ CDM galaxy formation.

INTRODUCTION

Very recently, a discovery seems to have blown up the fields of cosmology, astrophysics and elementary particles. The dark matter, the elusive substance that cosmologists believe to constitute about the 25% of the mass energy of the Universe and to play a crucial role in the birth and evolution of its structures, seems to have disappeared. In the dark, as someone is suggesting. For the past 30 years astrophysicists have believed that spiral galaxies were surrounded by dark halos ([1]) made by massive elementary particles that interact with the rest of the Universe (almost) only by gravitation. After a straightforward analysis of new accurate data, Mc Gaugh et al. (2016) claimed that this scenario faces a challenge. They found, from the kinematical and photometric data of a 153 spirals, that their radial accelerations show an anomalous feature: they correlate, at any radius and in any object, with their components generated only from the baryonic matter. This occurs in a very peculiar way which seems to lack for an immediate physical explanation and to be at variance with the presence, around spirals, of halos made by dark massive particles. Furthermore, they claimed that, in any case, this relationship proves that the eventual elementary particles making the dark halos cannot collisionless.

The aim of this paper is to show that, in the framework of Newtonian dark matter halos, the Mc Gaugh et al. (2016) relation is just a low resolution realization of a wellknown scenario which describes the *common* past history of the dark and the luminous components of Spirals.

In the present framework the equation of centrifugal equilibrium reads :

$$g(r) = g_h(r) + g_b(r) \quad (1)$$

g, g_h, g_b are, total radial acceleration and its components generated by the DM halo and by the baryonic matter, respectively. In detail, $g(r) = V^2(r)/r$, where $V(r)$ is the circular velocity and

$$g_b(r) = (V_D^2 + V_B^2 + V_{HI}^2)/r, \quad g_h(r) = V_H^2/r \quad (2)$$

the velocity fields V_i are the solutions of the four separated Poisson Equations for the dark and baryonic components: $V_i^2 = R d\Phi_i/dR$ and $\nabla^2 \Phi_i = 4\pi G \rho_i$, where ρ_i are dark matter, stellar disk, bulge, HI disk surface/volume densities) ($\rho_H, \rho_B, \mu_D(r)\delta(z), \mu_{HI}(r)\delta(z)$) with $\delta(z)$ the Kroenedeker function, z the cylindrical coordinate and Φ_i the gravitational potential. Of course, at any radius: $V^2 = (V_D^2 + V_B^2 + V_{HI}^2) + V_H^2$

In statistical studies we can assume that the main component of the luminous matter is the well known thin stellar disk with an exponential surface density profile:

$$\Sigma(R) = \Sigma_0 e^{-R/R_D} \quad (3)$$

where $\Sigma_0 = (M_d/L)I_0$ is the central surface mass density, with (M_d/L) the stellar disk mass to light ratio, I_0 the central surface brightness, L is the total luminosity in a specific band [4]. We will work in the I band that, in Spirals, well represents the stellar disk surface density. We define R_{opt} , the radius encompassing 83 % of the total luminosity/mass in stars, as the size of

the stellar disk. We have $R_{opt} = 3.2R_D$, R_{opt} is a quantity describing both the surface density profile and the stellar disk size. Observationally, we have, (e.g. [18]):

$$\log\left(\frac{R_D}{\text{kpc}}\right) = 0.633 + 0.379 \log\left(\frac{M_D}{10^{11}M_\odot}\right) + 0.069 \left(\log\frac{M_D}{10^{11}M_\odot}\right)^2, \quad (4)$$

that links the spiral's stellar disk masses and their sizes.

We remind that, in early Hubble-type spirals, at $0 \leq r < 0.2R_{opt}$, a central bulge dominates the circular velocity while, in the low luminosity, late Hubble types spirals, for $r > R_{opt}$, the gaseous HI disk gives a contribution to the circular velocity which is negligible with respect to that of the dark halo, but comparable with that of the stellar disk.

The converging outcome of two independent lines of investigation evidentiates that the dark and luminous mass distributions in spiral galaxies are universal specific functions of the disk mass M_D , (or of the disk Luminosity), and of the disk size R_{opt} [9, 11]. Thus, with the help of Eq(3) and with the disk mass (or the Luminosity) as the running variable in these functions, one obtains, for the entire family of spirals, the dark and the luminous matter structural parameters that straightforwardly lead to the corresponding g_h and g accelerations. Then, we can carefully investigate, with thousands of objects, the validity of the McGaugh relationship, whether it is compatible with the the dark matter halo scenario and, in such case, how weird is the relative DM particle.

Let us stress that the relationships in McGaugh et al. (2016) and in this work regard Spirals, in Ellipticals and in Dwarf spheroidals, , the determination of the above accelerations is very difficult due to their complex kinematics and dynamics and to their poorly known DM halo density profiles (see e.g. [5]), thus no safe relationship can be established.

In this work velocities are in km/s, and distances in kpc accelerations m/s^2

We proceed as it follows. In the second section we describe the Mc Gaugh et al. (2016) relationship, in the sections from the 3 to the 5 we work out such relationship by means of three different methods applied to 5 samples. The Implications and a discussion of the findings are put forward in sections 6 and 7. Let us notice that the Appendices A-C are a resume' of the results obtained in previous works that are used here.

THE MC GAUGH ET AL. (2016) RELATIONSHIP

McGaugh et al. (2016) investigated a sample of 153 galaxies across a large range of scales in luminosity and Hubble Types and having high quality (SPARC) rotation curves $V(r)$. For each object they derived the total radial acceleration $g(r)$ that compared with the corresponding value of the gravitational acceleration $g_b(r)$ generated, at the same radius r , by all the luminous matter of the galaxy. McGaugh et al. (2016) used the surface brightness and the color of each galaxy to derive their stellar surface/volume mass densities. These were inserted into the relative Poisson equation to provide them with the $g_b(r)$ values out to $(1 - 1.5) R_{opt}$, according to the extension of each RC. The systematics in method used is, at most 0.1 dex, and it has no effect on the relationship found. The entire procedure is sound: in this work, the McGaugh et al. (2016) determinations of $g_b(r)$ and $g(r)$ are not under discussion. The 1-sigma region of the Mc Gaugh et al. (2016) relationship is covered in Fig (1) by blue empty circles.

The strong correlation between the two acceleration reads as:

$$g(r) = g_b(r)/(1 - \text{Exp}[-(g_b(r)/a_0)^{0.5}]) \quad (5)$$

with $a_0 = 1.2 \times 10^{-10} m/s^2$.

At low g , g_b accelerations, this relation is clearly very different from that we would expect in the Newtonian no Dark Matter framework: $g(r) = g_b(r)$. In fact, in this case, the centrifugal acceleration must perfectly balance the gravitational acceleration due to the whole distribution of all the luminous baryons in the galaxy.

This result may pave the way for alternatives to the Dark Matter or to the Galileian Inertia Law eg [7]. However, in this paper we will focus on the Mc Gaugh et al. (2016) strong claim that the relationship in Eq.(5) is a game changer also within the Newtonian DM scenario. Thus, the issue in which we are interested concerns the assumption of the presence of dark matter halos around galaxies: does the above two-accelerations relationship challenges it ? Does it lead to a new understanding of internal dynamics of galaxies? Is it tantamount to the existence of elementary particles not yet considered as candidate for Dark Matter ? We will answer these questions in the next sections.

THE MCGAUGH ET AL. (2016) RELATION RELATION FROM THE UNIVERSAL ROTATION CURVE

We can represent the rotation curves (RCs) of late types Spirals by means of the Universal Rotation Curve (URC) pioneered in [2, 6] and set by [9] and [10]. In short, by adopting the normalized radial coordinate $x \equiv r/R_{opt}$, the RCs of Spirals are well

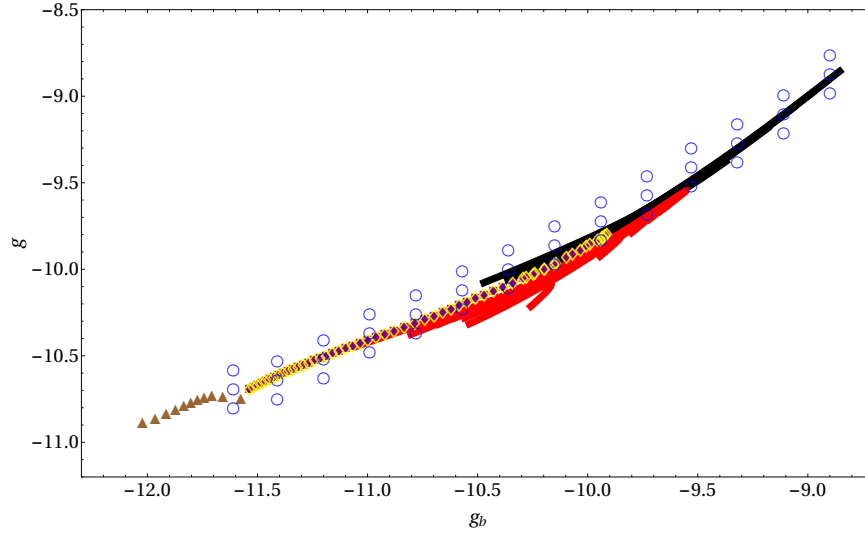


FIG. 1: The Mc Gaugh et al. relationship and its $1\text{-}\sigma$ uncertainties (blue circles), compared to the relationships obtained by means of the URC (red lines) and the RTF methods (black lines), in M 33 for Burkert and NFW halos (blue squares and yellow diamonds) and in NGC 3741 (brown triangles).

described by a Universal function of x and of M_I , their absolute I magnitude. This function, $V_{coadd}(x, M_I)$, has been derived by means of the analysis of about 1000 rotation curves (see Fig 1 of [9]) and supported by following investigations (see e.g. [12])). We can uniquely and successfully fit the V_{coadd} data by means of a universal analytical velocity model $V_{URC}(x, M_D)$ which includes the following velocity components: the standard Freeman disk $V_{URCD}(x, M_D)$, a cored dark matter halo $V_{URCH}(x, M_D)$ and a HI disk $V_{URCHI}(x, M_D)$ [19](see Appendix A for details).

Then, by means of the URC velocity model we can compute the above accelerations in a very large amount of objects, 5.2 times larger that of Mc Gaugh et al. (2016), and at radii congruent with those of McGaugh et al. (2016). We have, by writing now the accelerations explicitly as a function of r : $g_{URC}(r) = V_{URC}^2(x, M_D)/r$. The baryonic component of the radial acceleration is then obtained by removing from the latter the Dark Matter halo contribution $g_{hURC}(r) = V_{URCH}^2(x, M_D)/r$, so: $g_{bURC}(r) = g_{URC}(r) - g_{hURC}(r)$.

We plot in Fig (1) $g_{URC}(g_{bURC})$ for objects in the above I magnitude range and for radii $0 \leq r \leq 1.5 R_{opt}$ (red lines whose width indicates the fitting uncertainty).

THE MCGAUGH ET AL. (2016) RELATION RELATION FROM THE RADIAL TULLY FISHER RELATIONSHIP

Yegorova and Salucci (2007), by analyzing three different samples of 794, 86 and 91 spirals of different Hubble types that include a fair number of objects with a prominent bulge, discovered the Radial Tully-Fisher (RTF) relation (see [11]). This is a series of tight and independent relations between the galaxy absolute magnitude M_I and $\log V_n$, the circular velocity measured, in each object, at the same fixed fraction R_n/R_{opt} of the disk size R_{opt} . In detail, $R_n = (n/10) R_{opt}$, $1 \leq n \leq 10$, ($R_{opt} \equiv 3.2R_D$). In detail:

$$M_I = a_n \log V_n + b_n, \quad (6)$$

with a_n and b_n the parameters of the fits and n the indicator of the radial coordinate. Due to the limited number of optical kinematical data in the outermost regions of spirals, the RTF relationship is established only out to R_{opt} . All the 10 relationships in Eq. (6) are independent and statistically relevant [11]. The values of the a_n parameters are very similar in all the three samples and we have [11] :

$$a_n = -2.3 - 9.9(R_n/R_{opt}) + 3.9(R_n/R_{opt})^2 \quad (7)$$

The strong decrease of a_n with n in Eq. (7) implies that in Spirals the stellar light does not follow the distribution of the gravitating mass, which would require: $a_n = \text{const} \simeq -7.5$. As consequences, a) we must insert in the circular velocity model also a dark component, alongside the baryonic components a b) all velocity components must be function of (x, M_I) [11] (see Appendix B for details). Let us notice that the velocity model has also a bulge component and the assumed DM density

profile can account for cored and cusped DM halos. By best fitting the outcome of the RTF velocity model to the function $a_n(R_n)$ given in Eq. (7) we get the structural parameters of its components and, in turn: $V_{RTFD}(r/R_{opt}, M_I)$, $V_{RTFB}(r/R_{opt}, M_I)$, $V_{RTFH}(r/R_{opt}, M_I)$. Noticeably, the resulting best fit velocity model prefers a DM cored distribution, but, differently from the results of the URC method, also cuspy models lay within the fitting uncertainties. (see Appendix B for details).

Then, by means of Eq. (4) we can compute the above accelerations in three samples with large amount of objects, (4, 0.75, 0.70) times wider, than that of Mc Gaugh et al. (2016) et. , and at radii congruent with those of McGaugh et al. (2016). In detail: $g_{RTF}(r) = V_{RTF}^2(r)/r$ and $g_{bRTF}(r) = (V_{RTF}^2(r) - V_{RTFH}^2(r))/r$.

We plot in Fig. (1) $g_{RFT}(g_{bRFT})$ (black lines) by running the absolute magnitude M_I within the range specified above and for radii between $0.1 R_{opt}$ and R_{opt} .

THE MC GAUGH ET AL. (2016) RELATION FROM THE ACCURATE MASS DISTRIBUTION OF SPECIAL OBJECTS

In this section we work out the g_b vs g relation for two special, test-case spirals. We will reproduce their extended high quality RCs by means of a Individual Rotation Curve velocity model which includes a Freeman stellar disk, a HI disk and a dark halo. We will derive V_{IRCH} out to large galactocentric distances.

M33 is a low-luminosity galaxy important for investigating the distribution of dark matter in galaxies. In fact, it has no bulge, it is very rich in gas and, due to its proximity, its rotation curve has an excellent spatial resolution [8]. At outer radii, the HI disk is the major baryonic contribution to the galaxy circular velocity. Moreover, M33 is one of the fewest objects whose rotation curve is well reproduced both by a NFW cuspy and a cored DM halo density profile [8].

NGC 3741 is a dwarf spiral with the most extended available HI disk rotation curve in terms of the galaxy stellar disk size length scale, R_D [13]. This galaxy has been observed in the HI 21cm line with the Westerbork Synthesis Radio Telescope out to $42 R_D$ i.e. $13.5 R_{opt}$. The rotation curve, accurately derived from HI data cubes, is decomposed into its stellar, gaseous and dark components, the latter represented by a Burkert halo, necessary, in this object, for a successful fit. The best fit parameters for the IRC velocity model are shown in Appendix C.

Then, for these top cases of the investigation of dark matter in spirals, we directly obtain the radial accelerations $g(r) = V_{IRC}^2(r)/r$, while their baryonic contributions $g_b(r)$ are obtained by means of the differences between the radial accelerations and their Dark Matter components: $g_b(r) = (V_{IRC}^2(r) - V_{IRCH}^2(r))/r$.

Then, for the 2 objects, we get: $g = g_{IRC}(g_{bIRC})$ that we plot in figure (1) (as points).

DISCUSSION

In this work, within the Newtonian Gravity, we have investigated the $g_b - g$ relationship by McGaugh et al. (2016) in several large samples of rotation curves of spirals, spanning a very wide range in luminosities and Hubble types. In an approach complementary to theirs and with about ten times data than them, we have *assumed* the presence of dark matter halos as the origin of the “anomalies” of spiral’s kinematics in particular the relationship under study. In detail, by means of proper velocity models including, in all cases, a stellar disk and a DM halo, in two cases also a HI disk, and in one case also a stellar bulge, we have derived, by means of three different methods, the above accelerations. The first method we used comes from the phenomenology of spiral rotation curves and the inability of the luminous matter to match the circular velocity profiles without the assistance of additional specific dark halo component. This method is very sensitive to the distribution of the DM inside R_{opt} . The second method exploits the existence, at different normalized radii of spirals, of tight kinematics vs luminosity relationships, which yields the DM halo velocity profile. This method considers the stellar bulge component and it is very sensitive to the *amount* of DM inside R_{opt} . The third method is the standard mass modelling of extended high resolution rotation curves with optical and HI photometry which provides us with DM halo velocity profile. This method is sensitive also to the distribution of baryons well outside R_{opt} .

These 3 different methods were applied to 5 Samples for an amount of estimated (15500, 6500, 1500, 1500, 100) independent data, very much deepening the investigation by McGaugh et al. (2016) et al.

In all the cases, $g_b(r)$, the baryonic component of the radial acceleration is obtained by subtracting the DM halo contribution from the total radial acceleration $g_b(r) = g(r) - g_h(r)$. This component is so derived in a different and even opposite way with respect to that in Mc Gaugh et al. (2016) et al where, instead, it is obtained *directly* from the properties of the luminous matter with no involvement of the Dark Matter ones.

All our methods applied to our samples leads to a same universal $g = g(g_b)$ relationship, see Fig. (1). Its comparison with that found by Mc Gaugh et al. (2016) et al has striking implications. The $g(r)$ and $g_b(r)$ accelerations obtained by means of our velocity models in our samples lead to essentially the same relationship of Mc Gaugh et al. (2016), see Fig. (1). Let us stress again that our framework has a DM halo in every galaxy whose presence plays the crucial role of bending the expected

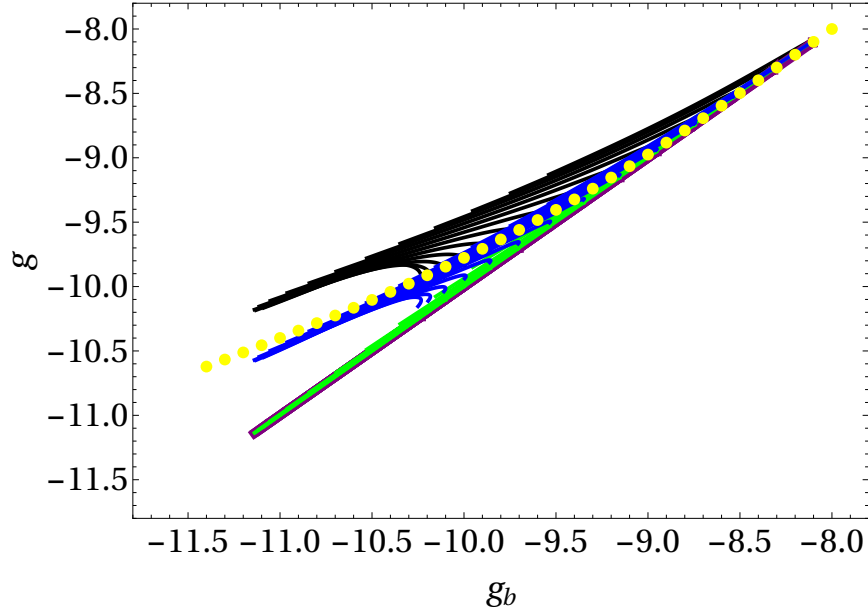


FIG. 2: The Mc Gaugh et al. (2016) relationship (yellow points) and those found in this work (blue lines). The relationships that emerge, for the same luminosities and radii, in the cases of -no dark matter (red lines), -compact dark matter (purple lines) - all dominating dark matter (black lines) and -increasing with luminosity fraction of DM (green lines), are also shown.

$g_b = g$ relation into one which results very consistent with that found by McGaugh et al. (2016) (see Fig. (1)) which is, instead, obtained in a way completely agnostic to the DM presence.

Therefore, considering the very good agreement of these two approaches, there are no reason for claiming that the Mc Gaugh et al. (2016) relation is alien to the scenario featuring DM halos in Galaxies or that it challenges it. On the contrary, any scenario which does not feature such dark halos is obliged to interpret our mirror relationships as repeated coincidences. Furthermore, in the present dark matter halos framework, the Mc Gaugh et al. (2016) relationship unravels no secrets about the dark halo component in Spirals. Specifically, it yields no relevant information about the DM halo density profile: in fact, we find that velocity models both with NFW and with a cored DM profile lay within the uncertainties of the Mc Gaugh et al. relationship. The intrinsic nature of the dark particle is beyond the physics underlined by such relationship.

The next step is to explain the Mc Gaugh et al. (2016) and ours relationship in terms of well understood general properties of dark matter. Let us assume a General velocity profile $V_G(x)$ and the relation in Eq. (4). For the stellar disk component $V_{GD}(x)$, we adopt the usual velocity profile of Eq. (10), for the HI disk contribution $V_{GHI}(x)$, the approach described by Eq. (11). Then, the dark halo component reads:

$$V_{GH}^2(x) = 2.65 \cdot 10^5 M_D / R_D^2 B x^d / (a + x^2) (M_D / (10^{11} M_\odot))^c \quad (8)$$

where B is proportional to the fractional content of dark matter at R_{opt} , c indicates the dependence of the latter quantity on the disk mass, a indicates the size of DM halo core and d indicates how compact is the distribution of dark matter with respect to that of the stars.

The General halo velocity model in Eq. (8) includes the models adopted in the previous sections, but it can represent also very different ones. Obviously, at any $x \equiv r/R_{opt}$, we have: $V_G^2 = V_{GH}^2 + V_{GD}^2 + V_{GHI}^2$. The disk mass is the running variable.

We have: $g_G(r) = V_G^2/r$ and: $g_{bG}(r) = (V_G^2 - V_{GH}^2 - V_{GHI}^2)/r$. Now we invert the process performed in the previous sections and we use the McGaugh et al. (2016) relationship data to get the four free parameters of the General model of spiral velocities. We found: $a = 1$, $c = -1/2$, $d = 2$, $B = 0.1$, with the formal fitting uncertainties on the parameters running from 20% to 50%. Noticeably, these best fit values for the parameters are very similar to those we have assumed in the three velocity models considered in the previous section (see Fig. (2)). This is expected in that the General model does include our velocity models that, in turn, yield the Mc Gaugh et al. (2016) relationship.

Let us investigate the situations in which the General model relationship *fails* to reproduce the Mc Gaugh et al. (2016) relationship. The quantity a plays no role in the agreement between these two relationships: we can take $0.4 < a < \infty$ without breaking it. Instead, for values of the quantity B such as: $B \approx 0$ (no dark matter) or $B > 0.3$, i.e. for an amount of DM > 3 times the best fit value, the agreement breaks down and the General model fails to reproduce the Mc Gaugh et al. (2016) (and ours) relationship (see Fig (2)). Similarly, the agreement continues also for values of d different from the best fit value of 2, but, for

$d < -2/3$, i.e. for a DM halo distributed in a more compact way than the luminous matter, the agreement breaks down (see Fig. (2)).

Therefore, in the Newtonian dark matter scenario, the Mc Gaugh et al. (2016) et relationship, enhanced by the results of this work, *exists* since and only since a) the luminous matter is more concentrated than the dark matter: the quantity $g_h(r)/g_b(r)$ increases with radius. b) in lower luminosity objects there is a larger fraction of dark matter: the quantity $g_h(R_{opt})/g_b(R_{opt})$ increases with decreasing galaxy luminosity.

Notice that:

1) a) This evidence was known since the very discovery of DM in spirals (e.g. [1]) and it comes from the most important property of the Dark Particles: they do interact with baryons only gravitationally. It is generally agreed that in protohalos the dark matter and the baryons were distributed in undifferentiated way, but during the following assembling of the stellar disks, the infalling baryons did dissipate much of their kinetical energy and fell deep inside into the galaxy potential well. The collisionless DM particles instead, conserved all their primordial kinetic energy and populated the outer parts of the halos.

2) b) This evidence is known since [16] and it is easily explained by the fact that the lower is the luminosity of a galaxy and then so its gravitational potential well, the more efficiently the energy injected into the interstellar space by Supernovae explosions has removed the neutral hydrogen from the galaxy, preventing it to be turned into stars.

3) a) and b) clearly emerge in most of related simulations and semi analytical studies ever performed in Λ CDM scenario.

CONCLUSIONS

There are strong views that the Mc Gaugh et al. (2016) relationship is a very special one. Infact, it connects two physical quantities measured at a same place in all objects and at all radii: the relation comes out independent of the galaxy magnitude, color, maximum circular velocity, central brightness, Hubble type, stellar disk lenghtscale, HI content and present star formation rate. Moreover, it allows an observer who measures her/his radial acceleration with respect to the center of the galaxy $g(r)$, to know, at the same time, the gravitational acceleration $g_b(r)$ due to all baryons of the galaxy s/he is subject to and this despite the evidence that, in galaxies, the light does not trace the gravitating mass. The claims that we are facing a meta-universal relationship, according to which, the distribution of dark matter is subjected to that of the luminous matter (or viceversa) seem justified.

Instead, the results of the present work imply that such view is just a mirage. Although the Mc Gaugh 2016 relationship is connected to physics, in that it arises from a combination of physical effects: the collisionless nature of the dark particle, the infall of baryons in the DM halos and the energy deposited into the interstellar space by Supernovae explosions, however, it is not directly related to a specific physical process, least of all it leads to new physics or to exotic particles.

Concluding, the presence of DM halos in galaxies is perfectly compatible with the McGaugh et al. (2016) relationship, that, obviously, cannot be considered as an evidence against the DM halos of elementary particles hypothesis. The McGaugh et al. (2016) relationship is reproduced by theoretical - numerical studies within the Λ CDM scenario [5] but yields no valuable information about the Dark Matter distribution in Spirals and the related intrinsic nature, differently from other galaxy scaling laws (e.g. see [12]). Then, in order to open the portal to new physics, we should head towards other evidences that dark and luminous matter behave in galaxies as active partners, and not as components which just share a common gravitational well, (e.g. [14], [21])

A 10-min video in which I will discuss this paper, available at: <https://www.youtube.com/watch?v=8K-VCoxJdus&t=20s>

APPENDIX A

V_{coadd} is obtained from the PS sample of 967 galaxies in the following way: 1. We divide the full spiral I magnitude range: $-23.5 \lesssim M_I \lesssim -17$ in 11 successive bins, each of them centered at M_I , as listed in [9] We assign each RC of the Sample to its corresponding luminosity bin. We coadd, in normalized coordinates $x = r/R_{opt}$, all the RCs assembled in each luminosity bin, and we average them to get $V_{coadd}(r/R_{opt}, M_I)/V_{coadd}(1, M_I)$ see Fig (1) of [9]

We fit these V_{coadd} data with the URC velocity model defined as.

$$V_{URC}^2(x, M_D) = V_{URCD}^2(x, M_D) + V_{URCH}^2(x, M_D) + V_{URCHI}^2(x, M_D) \quad (9)$$

the disk component is given by:

$$V_{URCD}^2 = \frac{GM_D}{2R_D} (3.2x)^2 (I_0 K_0 - I_1 K_1) \quad (10)$$

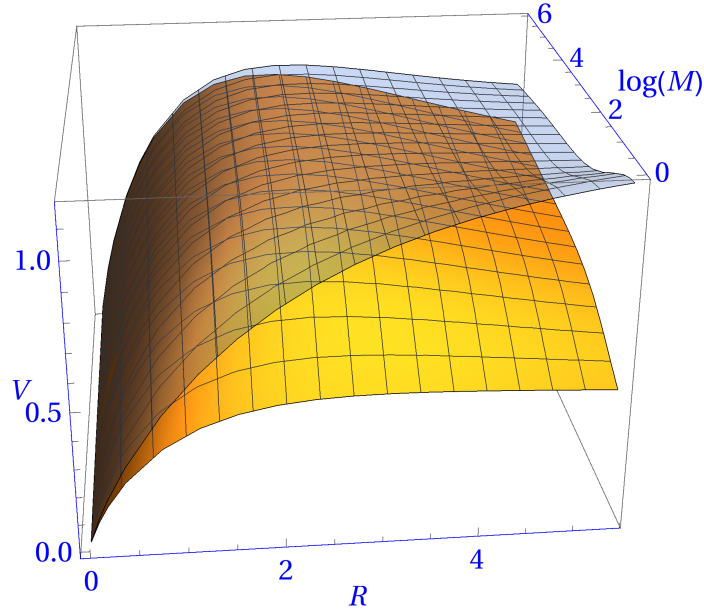


FIG. 3: $V(x, M)/V(1, M)$ and $V_b(x, M)/V(1, M)$ obtained by the URC. $\log(M_D/M_\odot) = 11 + 1/3 \log M$

with the Bessel functions evaluated at $1.6x$.

The HI component is modeled as [17, 18]: with an exponential thin disk of length scale $3R_D$

$$V_{URCHI}^2 = V_{URCD}(x) 10^7 (M_\odot/M_D)^{0.8} (1 + (M_D/(3.3 \times 10^{10} M_\odot))^{0.75}) \quad (11)$$

with the Bessel functions evaluated at $1.6/3x$.

For the dark halo we assume the Burkert profile density profile: $\rho_{URCH}(r) = \frac{\rho_0 r_0^3}{(r+r_0)(r^2+r_0^2)}$ with r_0 the core radius and ρ_0 the central halo density the free parameters, we have

$$V_{URCH}^2(r) = 6.4 \frac{\rho_0 r_0^3}{r} \left(\ln\left(1 + \frac{r}{r_0}\right) - \arctan \frac{r}{r_0} + \frac{1}{2} \ln\left(1 + \frac{r^2}{r_0^2}\right) \right). \quad (12)$$

we get :

$$\log(M_D/M_\odot) = -0.52M_I - 0.45 \quad (13)$$

$$\log \frac{\rho_0}{\text{g cm}^{-3}} = -23.515 - 0.964 \left(\frac{M_D}{10^{11} M_\odot} \right)^{0.31} \quad (14)$$

$$\log \left(\frac{r_0}{\text{kpc}} \right) \approx 0.66 + 0.58 \log \left(\frac{M_{\text{vir}}}{10^{11} M_\odot} \right) \quad (15)$$

that give us the URC dark halo component: $V_{URCH}(x, M_D)$ for $0 \leq r/R_{\text{opt}} \leq 2$. V_{URC} and $(V_{URC}^2 - V_{URCH}^2)^{1/2}$ obtained by this method at any radius and for any disk mass are shown in Fig. (3). We remind that $g_i = V_i^2/r$. The uncertainty in the estimate of the various accelerations is about $< 20\%$, i.e. negligible for the aim of this paper. Notice that $g_b(r) < g(r)$ in the region where the HI disk gives a contribution to $g_b(r)$ bigger than the disk contribution.

APPENDIX B

It is useful to define l as the fraction of a spiral I-band absolute luminosity with respect to $M_I = -24$, a reference magnitude corresponding to the most luminous spiral in our samples, $l = 10^{-(M_I+24)/2.5}$. The RTF velocity model has 3 components (disk,

bulge, halo) with: $V_{RTF}^2 = V_{RTFD}^2 + V_{RTFH}^2 + V_{RTFB}^2$. Let us notice since the RTF method uses relationships inside R_{opt} , we neglect in the velocity model the (very small) contribution to the circular velocity due to the HI disk.

For the stellar disk velocity component V_{RTFD} we assume the standard Freeman law: i.e. the RHS of eq (10) For the bulge we assume the Hernquist mass profile with an half mass radius $0.16R_{opt}$. The bulge mass is assumed to be a fraction $c_B l^{0.5}$ of the disk mass M_D ; the power law index 0.5 is suggested by the bulge-to disk vs total luminosity relation in spirals. Then

$$V_{RTFB}^2 = \frac{1.21 c_B l^{1.3} x}{(x + 0.1)^2} \quad (16)$$

The amplitude of the halo contribution to the circular velocity is related with that of the disk by means of 2 parameters: $V_{RTFH}^2(R_{opt}) = c_h (l)^{(k_h-0.5)} V_{RTFD}^2(R_{opt})$

For the halo velocity contribution we adopt the following profile:

$$V_{RTFH}^2 = c_h l^{(k_h-0.5)} \left(\frac{x^2}{x^2 + \alpha^2} \right) (1 + \alpha^2), \quad (17)$$

This profile, used in [9], can represent both Burkert halos, in which $\alpha > 1$ and NFW halos, in which $\alpha < 1/4$. The parameters c_h , k_h , c_b and α specify completely the velocity model. We obtain them by best reproducing the $a_n = a(R_n)$ relationship. We found $k_h = 0.79 \pm 0.04$, $c_b = 0.13 \pm 0.03$, $c_h = 0.13 \pm 0.06$, $\alpha = 1_{0.5}^{+1}$ in units of R_{opt} . The uncertainties on the LM contributions to the circular velocity are $< 30\%$, i.e. negligible for the aim of this paper. V_{RTF} and $(V_{RTF}^2 - V_{RTFH}^2)^{1/2}$ obtained by the RTF method at any radius and for any luminosity are shown in Fig. (4). We remind that: $g_i = V_i^2/r$

Let us stress: 1) The RTF method deals with the bulge component in spirals and it shows that this component, as we can predict, has its play in the relationship at high accelerations when $g = g_b$ 2) The best fit value for α does not well discriminate, within its uncertainty, whether the DM density halos are cuspy or cored. Therefore the $g_{RTF}(g_b RTF)$ relationship, plotted with its uncertainties in Fig (1), represents both cored and cusped DM density models.

APPENDIX C

For an individual rotation curve, at any radius:

$$V_{IRC}^2 = V_{IRCD}^2 + V_{IRCH}^2 + V_{IRCHI}^2 \quad (18)$$

M 33 Notice that its stellar disk surface density has been derived from multi-band optical imaging and the correlations between galaxy colors and stellar mass-to-light ratios ([8]). The structural best fit values are in the NFW halo case : concentration: $c = 9.5 \pm 1.5$, halo virial mass: $(4.3 \pm 1.0) \times 10^{11} M_\odot$ and disk mass: $(4.8 \pm 0.6) \times 10^9 M_\odot$ and, in the URC halo case: core radius: $r_0 = (7.5 \pm 1.5) kpc$, central density: $\rho_0 = (1.8 \pm 0.3) \times 10^{-2} M_\odot / pc^3$ and disk mass $M_D = (7.3 \pm 0.6) \times 10^9 M_\odot$.

NGC 3741 The structural best fit values are: $r_0 = (3 \pm 0.5) kpc$, $\rho_0 = (1.6 \pm 0.3) \times 10^{-24} M_\odot / pc^3$ and $M_D(3.4 \pm 1.2) \times 10^7 M_\odot$. The uncertainties are reported for completeness they have no role in the results of this paper.

Notice that M 33 is a clear case in which, although the best fit $V_{IRCH}(R/R_{opt})$ is remarkably different according to that we force the halo density to a URC-cored or a to a NFW-cuspy profile, g_h , the halo contribution to the radial acceleration, takes similar values in the two different cases.

-
- [1] Rubin, V.C., Science 1983, vol. 220, 1339
 - [2] Rubin, V. C., Ford, W. K., Jr., Thonnard, N., & Burstein, D. 1982, ApJ, 261, 439
 - [3] McGaugh, S, Lelli, F., Schombert, J. , Phys. Rev. Lett. 2016, 117, 201101
 - [4] Freeman, K. C., 1970, ApJ 160, 811
 - [5] Fattahi, A; Navarro, J F.; Sawala, T., Frenk, el al 2016, arXiv160706479
 - [6] Persic, M & Salucci, P. 1991, ApJ, 368, 60.
 - [7] Milgrom, M., 1983, ApJ 270, 365
 - [8] Corbelli, E; Thilker, D; Zibetti, S; Giovanardi, C; Salucci, P. 2014, A&A, 572, 23
 - [9] Persic, M., Salucci, P., Stel, F., 1996, MNRAS 281, 27
 - [10] Salucci, P.; Lapi, A.; Tonini, C.; Gentile, G.; Yegorova, I.; Klein, U. 2007, MNRAS, 378, 41
 - [11] Yegorova, I, Salucci., P. 2007, MNRAS.377, 507
 - [12] de Laurentis , M, Salucci, P, Invited Review at VIII International Workshop on the Dark Side of the Universe, 2012, PoS(DSU 2012) 012
 - [13] Gentile, G., Salucci, P. Klein, U. Granato, G. L., 2007, MNRAS, 375, 199

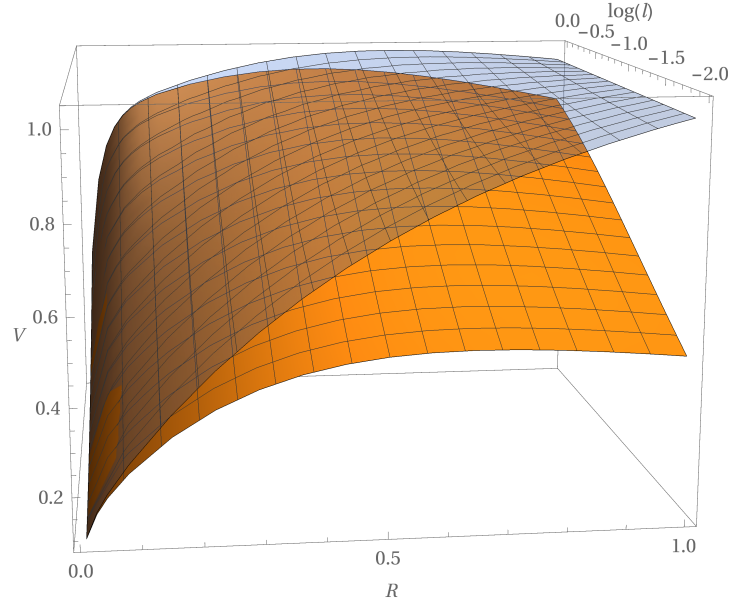


FIG. 4: $V(x, l)/V(1, l)$ and $V_b(x, l)/V(1, l)$ obtained by the RTF method. $\log l = (M_l - 23.5)/5$

- [14] Donato, F., Gentile, G., Salucci, P., 2004, MNRAS, 353L, 17
- [15] Gentile G., Salucci P., Klein U., Vergani D., Kalberla P., 2004, MNRAS, 351, 903
- [16] Persic M., Salucci P., 1988, MNRAS, 234, 131
- [17] Evoli, C., Salucci, P., Lapi, A., & Danese, L. 2011, ApJ, 743, 45
- [18] Tonini, C.; Lapi, A.; Shankar, F.; Salucci, P. 2006, ApJ, 638, 13
- [19] Salucci, P., Burkert, A. 2000, ApJ, 5379
- [20] Karukes, E. V., & Salucci, P. 2016, arXiv:1609.06903
- [21] Gentile, G., Famaey, B., Zhao, H., & Salucci, P. 2009, Nature, 461, 627